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CONVECTION OF A FERROFLUID IN AN ALTERNATING MAGNETIC FIELD

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Parametric convective instability of a horizontal layer of a homogeneous ferrofluid under the action of an alternating magnetic field is studied. A case with rigid boundaries is considered. Convection thresholds are found. In an alternating magnetic field with a zero mean value, perturbations are found to have a synchronous character. These perturbations, however, can belong to different classes, because they depend on the temperature difference on the layer boundaries, the layer thickness, the frequency and amplitude of the alternating external field, and the physical properties of the ferrofluid.

Key words: *ferrofluid, thermomagnetic instability, parametric convection.*

Introduction. Magnetic fluids or ferrofluids are colloid solutions containing magnetic nanoparticles covered by a surfactant for preventing their aggregation and suspended in a non-conducting fluid. Systematic investigations of ferrofluids were started in [1, 2], where mechanical equilibrium of a nonisothermal ferrofluid in a magnetic field was found to be impossible.

The thermomagnetic mechanism of ferrofluid instability in an external magnetic field is determined by inhomogeneity of its magnetization under nonuniform heating and is manifested even under microgravity conditions. The effect of thermomagnetic convection is based on the dependence of magnetization on temperature: under identical conditions, magnetization of the colder element of the fluid is greater; hence, this element experiences the action of a greater magnetic force in the direction of the magnetic field gradient [3]. There are numerous theoretical and experimental studies of convection of ferrofluids, in particular, stability thresholds and evolution of stationary and wave structures in these fluids (see, e.g., [4–6]). It was demonstrated in these investigations that interaction of thermomagnetic and Rayleigh instability induces complicated convection regimes and affects the heat transition through the layer. (Such properties of convection in ferrofluids can be used in devices to monitor heat transfer and control heat fluxes.) The influence of alternating magnetic fields on convection in ferrocolloids has been less studied (see, e.g., [7–9]), despite the fact that the presence of a modulated parameter exerts a significant effect on convective stability of equilibrium and fluid flow. Parametric excitation of convection in a ferrofluid was studied under conditions where the ferrofluid had a homogeneous composition [7, 9] or was stratified in a nonuniform magnetic field [8]. It was demonstrated that field modulation can substantially decrease the convection threshold owing to parametric instability. It should be noted that Kaloni and Lou [9] considered convective instability of a magnetic fluid in magnetic field, but they did not study the character of the response of the convective system under the action of an alternating magnetic field and did not consider the properties of various regimes of parametric convection. In the present work, the effect of an alternating uniform magnetic field on convection in a horizontal layer of a ferrofluid is studied within the framework of a quasi-stationary approach, convection thresholds in the plane "amplitude – inverse frequency of the external field" are found, and various types of the ferrofluid response to external actions are considered.

1. Formulation of the Problem. Let us consider a horizontal layer of thickness h of a homogeneous ferrofluid. The layer is located in an external uniform magnetic field orthogonal to the layer. The magnetic field is varied by the law

$$\boldsymbol{H}^{\text{ext}} = H_0^{\text{ext}} \cos(\Omega t)\, \boldsymbol{e},$$

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where Ω is the cyclic frequency of field alternation, $\boldsymbol{e} = (0, 0, 1)$ is the unit vector orthogonal to the layer, $\boldsymbol{H}^{\text{ext}}$ is the strength of the external magnetic field, and H_0^{ext} is the amplitude of the strength of the external magnetic field. The origin of the coordinate system is located in the middle of the layer, the x axis is aligned in the plane of the layer, and the z axis is orthogonal to the layer surface and is directed upward. A constant temperature T_1 is maintained on the lower boundary of the layer, and the temperature of the upper boundary is T_2. The behavior of an incompressible non-conducting ferrofluid in the magnetic field is described by the following system of equations [3]:

$$\rho\,\frac{D\boldsymbol{v}}{Dt} = -\nabla p_H + \eta\,\Delta\boldsymbol{v} + \rho\boldsymbol{g} + \nabla\cdot\boldsymbol{H}\boldsymbol{B},$$

$$\Big[\rho C_{V,H} - \mu_0\boldsymbol{H}\cdot\Big(\frac{\partial\boldsymbol{M}}{\partial T}\Big)_{V,H}\Big]\frac{DT}{Dt} + \mu_0 T\Big(\frac{\partial\boldsymbol{M}}{\partial T}\Big)_{V,H}\cdot\frac{D\boldsymbol{H}}{Dt} = \varkappa\,\Delta T, \tag{1}$$

$$\operatorname{div}\boldsymbol{v} = 0.$$

Here ρ is the ferrofluid density, $D/Dt = \partial/\partial t + \boldsymbol{v}\cdot\nabla$, $\boldsymbol{v}$ is the velocity, $p_H = p + \boldsymbol{B}\cdot\boldsymbol{H}/2$, η is the dynamic viscosity, $\boldsymbol{g} = -g\boldsymbol{e}$ is the free-fall acceleration, $\boldsymbol{H}$ is the magnetic field inside the layer, $\boldsymbol{M}$ is the magnetization of the fluid element, $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$ is the magnetic induction, μ_0 is the magnetic constant, $C_{V,H}$ is the specific heat at constant volume and magnetic field strength, $\varkappa$ is the thermal conductivity, and T is the temperature. System (1) is supplemented with the Maxwell equations

$$\operatorname{div}\boldsymbol{B} = 0, \qquad \operatorname{rot}\boldsymbol{H} = 0. \tag{2}$$

The fluid velocity on the boundaries is equal to zero. The ferrofluid density is assumed to be a linear function of temperature:

$$\rho = \rho_0(1 - \alpha(T - T_a)) \tag{3}$$

(α is the coefficient of thermal expansion and T_a is the mean temperature in the layer).

The quasi-stationary model used [3, 9] implies that the magnetization vector field at each time instant acquires an equilibrium (in the thermodynamic sense) value, and the magnetization direction coincides with the magnetic field direction. In the general case, magnetization depends on the magnetic field and temperature in accordance with the Langevin law

$$\boldsymbol{M} = \frac{\boldsymbol{H}}{H}\,M_S L(\xi), \qquad L(\xi) = \coth\,(\xi) - \frac{1}{\xi}, \qquad \xi = \frac{\mu_0 m H_0}{k_{\text{B}} T_a}, \tag{4}$$

where M_S is the saturation magnetization, m is the magnetic moment of one particle, and k_{B} is the Boltzmann constant.

The state of mechanical equilibrium of the system is characterized by the following relations:

$$\begin{gathered}\boldsymbol{v}_{qs} = 0, \qquad T_{qs}(z) = T_1 - (z + h/2)A, \qquad A = (T_1 - T_2)/h,\\ \boldsymbol{H}_{qs} = H_{qs}\boldsymbol{e} = H_0\cos{(\Omega t)}\,\boldsymbol{e} = (H_0^{\text{ext}}/\mu)\cos{(\Omega t)}\,\boldsymbol{e},\\ \boldsymbol{M}_{qs} = M_{qs}\boldsymbol{e}, \qquad M_{qs} = M_S L(\xi),\\ \frac{dH_{qs}}{dz} = -\frac{dM_{qs}}{dz} = -\frac{1}{\mu}\Big(\frac{\partial M_{qs}}{\partial T}\Big)_H\frac{dT_{qs}}{dz}, \qquad \mu = 1 + \Big(\frac{\partial M_{qs}}{\partial H}\Big)_T.\end{gathered} \tag{5}$$

Let us consider small perturbations of the ground state (5):

$$\{\boldsymbol{v}, T, p_H, \boldsymbol{H}, \boldsymbol{M}, \boldsymbol{B}\} = \{0, T_{qs}, (p_H)_{qs}, \boldsymbol{H}_{qs}, \boldsymbol{M}_{qs}, \boldsymbol{B}_{qs}\} + \{\boldsymbol{v}, \theta, p', \boldsymbol{H}', \boldsymbol{M}', \boldsymbol{B}'\}.$$

In accordance with Eq. (2), the perturbations of the strength and induction of the magnetic field are presented as

$$\boldsymbol{H}' = \nabla\Phi\,; \tag{6}$$

$$\boldsymbol{B}' = \mu_0\Big\{\Big(1 + \frac{M_{qs}}{H_{qs}}\Big)\boldsymbol{H}' + \boldsymbol{e}\Big[\Big(\frac{\partial M_{qs}}{\partial T}\Big)_H\theta + \Big(\Big(\frac{\partial M_{qs}}{\partial H}\Big)_T - \frac{M_{qs}}{H_{qs}}\Big)H'_z\Big]\Big\}. \tag{7}$$

With allowance for Eqs. (3) and (4), the linearized system of equations for the perturbations takes the form

$$\rho_0 \frac{\partial \boldsymbol{v}}{\partial t} = -\nabla p' + \eta\,\Delta \boldsymbol{v} + \rho_0 \alpha \theta g \boldsymbol{e} + (\boldsymbol{B}_{qs} \cdot \nabla)\boldsymbol{H}' + (\boldsymbol{B}' \cdot \nabla)\boldsymbol{H}_{qs},$$

$$\frac{\partial \theta}{\partial t} + \boldsymbol{v} \cdot \nabla T_{qs} = K_\theta\,\Delta\theta, \qquad K_\theta = \frac{\varkappa}{\rho_0 C_{V,H}}, \qquad \operatorname{div} \boldsymbol{B}' = 0. \tag{8}$$

Taking into account Eqs. (6) and (7) and eliminating the pressure from Eq. (8), we obtain the equation for the perturbations of vertical velocity

$$\frac{\partial}{\partial t}\,\Delta v_z = \frac{\eta}{\rho_0}\,\Delta\Delta v_z + \alpha g \Delta_1 \theta - \frac{3\mu_0 A H_{qs} \chi_0}{\rho_0 T_{qs}}\,L'\,\Delta_1 \frac{\partial \Phi}{\partial z} + \frac{9\mu_0 A H_{qs}^2 \chi_0^2}{\rho_0 T_{qs}^2 (1 + 3\chi_0 L')}\,(L')^2\,\Delta_1 \theta,$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$, $\Delta_1 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $\chi_0 = \mu_0 M_S m/(3k_\mathrm{B}T)$ is the initial magnetic susceptibility, and $L' = dL/d\xi$.

We write the equation for the magnetic potential in the form

$$\Delta\Phi + \frac{M_{qs}}{H_{qs}}\,\Delta_1 \Phi + 3\chi_0 L' \frac{\partial^2}{\partial z^2}\,\Phi - 3\chi_0 L' \Big(\frac{A H_{qs}}{T_{qs}^2}\,\theta + \frac{H_{qs}}{T_{qs}}\,\frac{\partial \theta}{\partial z}\Big) = 0,$$

where

$$\frac{H_{qs}}{T_{qs}} \simeq \frac{H_0 \cos \Omega t}{T_a}, \qquad \frac{M_{qs}}{H_{qs}} = \frac{3\chi_0 L}{\xi}.$$

We introduce dimensional quantities, using the length h, the time $h^2\rho_0/\eta$, the temperature Ah, the velocity K_θ/h, and the magnetic potential hH_0 for normalization. Then, we obtain the following system of equations in the dimensionless form for normal perturbations periodic in the plane of the layer $\{v_z, \theta, \Phi\} \sim \exp\,(ik_x x + ik_y y)$:

$$\Big(\frac{\partial^2}{\partial z^2} - k^2\Big)\frac{\partial}{\partial t}\,v_z(z,t) = \Big(\frac{\partial^2}{\partial z^2} - k^2\Big)^2 v_z(z,t)$$

$$-\,\mathrm{R}(1 + M_1 \cos^2 \omega t) k^2 \theta(z,t) + \mathrm{R} M_2 \cos\,(\omega t) k^2\,\frac{\partial}{\partial z}\,\Phi(z,t),$$

$$\mathrm{Pr}\,\frac{\partial}{\partial t}\,\theta(z,t) - v_z(z,t) = \Big(\frac{\partial^2}{\partial z^2} - k^2\Big)\theta(z,t), \tag{9}$$

$$\frac{\partial^2}{\partial z^2}\,\Phi(z,t) - k_1^2 \Phi(z,t) - \frac{3\chi_0 L'}{1 + 3\chi_0 L'}\,M_3 \cos\,(\omega t)\,\frac{\partial}{\partial z}\,\theta(z,t) = 0.$$

Here the quantities $M_1 = \mu_0 A (3\chi_0 L')^2 H_0^2/(\rho_0 \alpha g \mu T_a^2)$ and $M_2 = 3\chi_0 L' \mu_0 H_0^2 (\rho_0 \alpha g T_a h)$ characterize the thresholds of thermomagnetic convection, $M_3 = Ah/T_a$, $k^2 = k_x^2 + k_y^2$, $k_1^2 = k^2(1 + (3\chi_0 L)/\xi)/(1 + 3\chi_0 L')$, $\mu = 1 + 3\chi_0 L'$, $\mathrm{R} = \rho_0 \alpha g A h^4/(\eta K_\theta) = \rho_0^2 C_{V,H} \alpha g A h^4/(\eta\varkappa)$ is the thermal Rayleigh number, $\mathrm{Pr} = \eta/(\rho_0 K_\theta) = \eta C_{V,H}/\varkappa$ is the thermal Prandtl number, and $\omega = h^2 \rho_0 \Omega/\eta$ is the dimensionless frequency.

In the case of rigid isothermal boundaries of the layer, system (9) is supplemented by standard boundary conditions

$$z = \pm\frac{1}{2}: \qquad v_z = \frac{\partial v_z}{\partial z} = \theta = 0, \qquad (1 + 3\chi_0 L')\,\frac{\partial}{\partial z}\,\Phi(z,t) \pm k\Phi(z,t) = 0. \tag{10}$$

To solve the amplitude problem (9), (10), we use the Galerkin method and approximate the vertical velocity by the even function

$$v_z(z,t) = w(t)(\cos\,(\pi z))^2;$$

we also expand the temperature perturbation $\theta(z,t)$ into a series with respect to even functions of z:

$$\theta(z,t) = \sum_{n=0}^{\infty} A_n(t) \cos\,(\lambda_n z), \qquad \lambda_n = \pi + 2\pi n. \tag{11}$$

The reason for these actions is the fact that the even mode of velocity perturbations is the most dangerous one in problems of thermal convection, where the perturbations can be divided into classes of different evenness [10].

Substituting Eq. (11) into the equation for the magnetic potential from system (9) and solving it in a standard manner with the boundary conditions for $\Phi(z,t)$, we obtain

$$\Phi(z,t) = \sum_{n=0}^{\infty} \varphi_n(t)\, \frac{3\chi_0 L' M_3}{1+3\chi_0 L'}$$

$$\times \left(\sin(\lambda_n z) + \frac{(-1)^{n+1} \sinh(k_1 z)}{\sqrt{(1+3\chi_0 L')(1+3\chi_0 L/\xi)}\, \cosh(k_1/2) + \sinh(k_1/2)} \right), \tag{12}$$

$$\varphi_n(t) = A_n(t) \cos(\omega t) \lambda_n/(k_1^2 + \lambda_n^2).$$

Note that solution (12) is an odd function of z. Moreover, it follows from Eq. (12) that the perturbations of the magnetic potential and temperature in time should have different time periods.

After orthogonalization, the system of equations for the amplitudes of the normal perturbations of velocity and temperature acquires the form

$$\left(\frac{\pi^2}{2} + \frac{3k^2}{8}\right) \frac{dw}{dt} = -w\left(2\pi^2 + \pi^2 k^2 + \frac{3k^4}{8}\right)$$

$$+ (\mathrm{R} + N\cos^2(\omega t))k^2 \sum_{n=0}^{n'} \frac{4A_n(-1)^{n+1}k_1^2}{\pi(8n^3+12n^2-2n-3)(k_1^2+\lambda_n^2)} - Nk^2\cos^2(\omega t)\, \frac{4\pi^2 \sinh(k_1/2)}{k_1^2+4\pi^2}$$

$$\times \sum_{n=0}^{n'} \frac{A_n \lambda_n (-1)^{n+1}}{(k_1^2+\lambda_n^2)\left(\sqrt{(1+3\chi_0 L')(1+3\chi_0 L/\xi)}\, \cosh(k_1/2) + \sinh(k_1/2)\right)}; \tag{13}$$

$$\mathrm{Pr}\, \frac{dA_m}{dt} = \frac{8(-1)^{m+1} w}{\pi(8m^3+12m^2-2m-3)} - A_m(k^2+\lambda_m^2), \qquad m = 0,1,2,\ldots,n', \tag{14}$$

where $N = \mathrm{R}M_1 = \mu_0 A^2 h^4 (3\chi_0 L')^2 H_0^2/[\eta K_\theta T_a^2 (1+3\chi_0 L')]$ is the magnetic Rayleigh number.

2. General Properties of Perturbations and Method of Solution. To analyze the general properties of perturbations, we write the first and second equations of system (9) in the matrix form as

$$B\, \frac{d}{dt} X(t) = A(t)\, X(t), \tag{15}$$

where

$$X(t) = [v_z(z,t), \theta(z,t)]^{\mathrm{t}}, \qquad B = \begin{pmatrix} D_z^2 - k^2 & 0 \\ 0 & \mathrm{Pr} \end{pmatrix}, \qquad D_z \equiv \frac{\partial}{\partial z},$$

$$A(t) = \begin{pmatrix} a_{11} & a_{12}(t) \\ a_{21} & a_{22} \end{pmatrix} = A(t + T_{\min})$$

is a quadratic matrix periodic in time with the fundamental period $T_{\min} = \pi/\omega$. The elements of the matrix $A(t)$ have the following form:

$$a_{11} = (D_z^2 - k^2)^2, \qquad a_{21} = 1, \qquad a_{22} = D_z^2 - k^2,$$

$$a_{12}(t) = -k^2\left[\mathrm{R} + N\cos^2(\omega t)\left(1 - \frac{3\chi_0 L'}{1+3\chi_0 L'}\, D_z \Xi\right)\right].$$

The operator Ξ is found from the relation

$$\Xi\theta(z,t) = \Phi_1(z,t),$$

where $\Phi_1(z,t)$ is the solution of the differential equation $D_z^2\Phi_1 - k_1^2\Phi_1 - D_z\theta = 0$ [with the boundary conditions (10)].

Let us consider a periodic solution of the form

$$X(t+\pi/\omega) = D\,X(t), \tag{16}$$

where D is a translational matrix (class matrix) determining the time symmetry and the properties of the considered type of the system response to a parametric action. Each of the four matrices

$$D_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad D_2 = -D_1 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$D_3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \equiv I, \qquad D_4 = -D_3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

corresponds to a specific type of the response simultaneous with respect to the external field with the period $2\pi/\omega$. Not all of these matrices, however, are realized in the convective system under study.

For all matrices $D = D^{-1}$ considered, Eq. (16) is written in the form

$$X(t) = DX(t+\pi/\omega). \tag{17}$$

Substituting Eq. (17) into Eq. (15), we obtain

$$\begin{aligned} BD\,\frac{d}{dt}\,X\Big(t+\frac{\pi}{\omega}\Big) &= A(t)DX\Big(t+\frac{\pi}{\omega}\Big), \\ \frac{d}{dt}\,X\Big(t+\frac{\pi}{\omega}\Big) &= D(B^{-1}A(t))DX\Big(t+\frac{\pi}{\omega}\Big). \end{aligned} \tag{18}$$

Comparing Eq. (18) with the result obtained from Eq. (15) after the external action on the system during a time period equal to one half of the period of the external magnetic field,

$$\frac{d}{dt}\,X\Big(t+\frac{\pi}{\omega}\Big) = B^{-1}A\Big(t+\frac{\pi}{\omega}\Big)\,X\Big(t+\frac{\pi}{\omega}\Big),$$

we obtain the condition of realization of one of the classes of synchronous perturbations corresponding to the matrices D_l ($l = \overline{1,4}\,$):

$$B^{-1}A(t+\pi/\omega) = D(B^{-1}A(t))D. \tag{19}$$

We can easily show that the classes of the matrices D_1 and D_2 cannot be realized in the system, because condition (19) is not satisfied for them. Writing the matrix B in the form

$$B = \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix},$$

we calculate the left side of Eq. (19)

$$B^{-1}A\Big(t+\frac{\pi}{\omega}\Big) = \frac{1}{b_{11}b_{22}}\begin{pmatrix} a_{11}b_{22} & a_{12}(t+\pi/\omega)\,b_{22} \\ a_{21}b_{11} & a_{22}b_{11} \end{pmatrix}$$

and also its right side

$$D_1(B^{-1}A(t))D_1 = \frac{1}{b_{11}b_{22}}\begin{pmatrix} a_{11}b_{22} & -a_{12}(t)b_{22} \\ -a_{21}b_{11} & a_{22}b_{11} \end{pmatrix}.$$

A comparison of these expressions shows that $B^{-1}A(t+\pi/\omega) \neq D_1(B^{-1}A(t))D_1$, because $a_{21}b_{11} \neq -a_{21}b_{11} \neq 0$ and $a_{12}(t+\pi/\omega)b_{22} \neq -a_{12}(t)b_{22}$. At the same time, the matrices D_3 and D_4 satisfy Eq. (19) identically.

It should be noted that the full convective system (9) is characterized by perturbations v_z, θ, and Φ; the temperature and magnetic potential perturbations, as was noted above, vary with different periods. Thus, two classes of synchronous (with respect to the external field) perturbations are realized in the system; these perturbations vary in time as follows:

$$t \to t+\pi/\omega\colon \qquad \begin{aligned} &v_z \to v_z, \quad \theta \to \theta, \quad \Phi \to -\Phi, \\ &v_z \to -v_z, \quad \theta \to -\theta, \quad \Phi \to \Phi. \end{aligned} \tag{20}$$

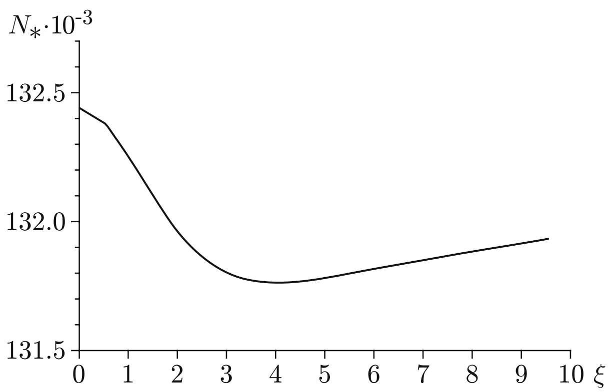

Fig. 1

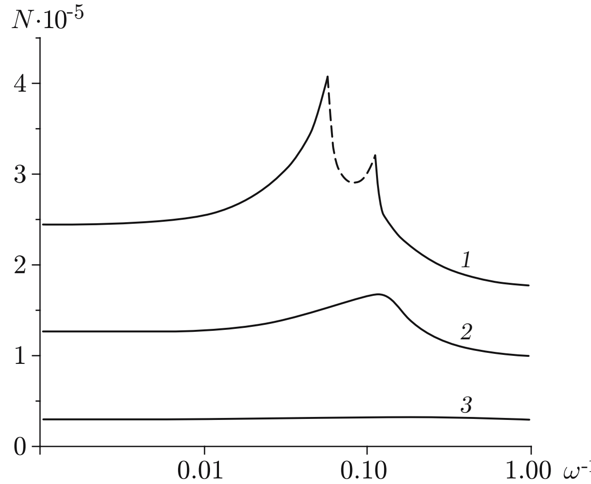

Fig. 2

Fig. 1. Critical magnetic Rayleigh number N_* versus the Langevin parameter ξ in a constant magnetic field at $\mathrm{R} = -108{,}200$ and $\chi_0 = 0.12$.

Fig. 2. Neutral curves for $\chi_0 = 0.12$, $\mathrm{Pr} = 25$, and different values of R: curve 1 refers to $\mathrm{R} = -100{,}000$, $\xi = 4.06$, and $k = 8.128$, curve 2 to $\mathrm{R} = -70{,}000$, $\xi = 4.10$, and $k = 7.447$, and curve 3 to $\mathrm{R} = -10{,}000$, $\xi = 4.45$, and $k = 5.066$; the solid and dashed curves show the stability boundaries for synchronous neutral perturbations of the classes H1 and H2, respectively.

We denote the first class (corresponding to the matrix D_3) and the second class (corresponding to the matrix D_4) by H1 and H2, respectively.

The search for the boundaries of convective instability and the analysis of the system response are performed on the basis of the Floquet theory [11]: by constructing the monodromy matrix for the amplitudes of the normal perturbations by means of numerical integration of system (13), (14) by the Runge–Kutta–Fehlberg method of the fourth or fifth order of accuracy and by calculating the eigenvalues, which are multipliers γ_k. Numerical integration of equations is performed on a time interval corresponding to the external field period $T_{\min} = 2\pi/\omega$. If the multipliers $|\gamma_1| \geqslant |\gamma_2| \geqslant \ldots \geqslant |\gamma_{n'+2}|$ are ordered, then the condition $|\gamma_1| = 1$ corresponds to the threshold of convective instability. The value of the multiplier is $\gamma_1 = 1$ in the case of neutral synchronous perturbations and $\gamma_1 = -1$ in the case of the subharmonic response threshold.

The calculations were performed for a ferrofluid consisting of magnetic particles with a diameter $d = 10$ nm, which were suspended in kerosene ($\mathrm{Pr} = 25$). The saturation magnetization was $M_S = 48{,}000$ A/m, and the initial magnetic susceptibility was $\chi_0 = 0.12$.

3. Thresholds of Parametric Instability. Let us consider the case with a constant external field ($\omega = 0$) and determine the critical values of the Langevin parameter ξ and the wavenumber k. For a fixed Rayleigh number R, the magnetic Rayleigh number $N = N(k, \xi)$ is a function of two variables. For a fixed value of ξ, the minimum $N = N_*$ on the curve corresponds to the critical value of the wavenumber k_*. The quantity N_* reaches the minimum value at $\xi = \xi_*$ (Fig. 1). The thus-obtained values of $k = k_*$ and $\xi = \xi_*$ are used in further calculations of the convection thresholds in an alternating field (at $\omega \neq 0$). In particular, $k_* = 8.129$ and $\xi_* = 4.12$ for $\mathrm{R} = -108{,}200$. The algorithm described above is used to calculate the convection thresholds for all values of the Rayleigh numbers R considered. The calculations show that the critical wavenumber k_* increases with increasing intensity of heating of the layer from above ($|\mathrm{R}|$ increases, $\mathrm{R} < 0$).

Let us consider the case with an alternating field. No subharmonic perturbations were found in the numerical study of stability. Figure 2 shows the neutral curves $N(\omega^{-1})$ (magnetic Rayleigh number vs. inverse frequency) for different values of the Rayleigh number R. The zones of convective instability lie above the curves. The calculation results confirm the validity of separation of synchronous perturbations into two classes. The effect of the parametric resonance is manifested only at sufficiently large (in absolute value) negative values of the Rayleigh

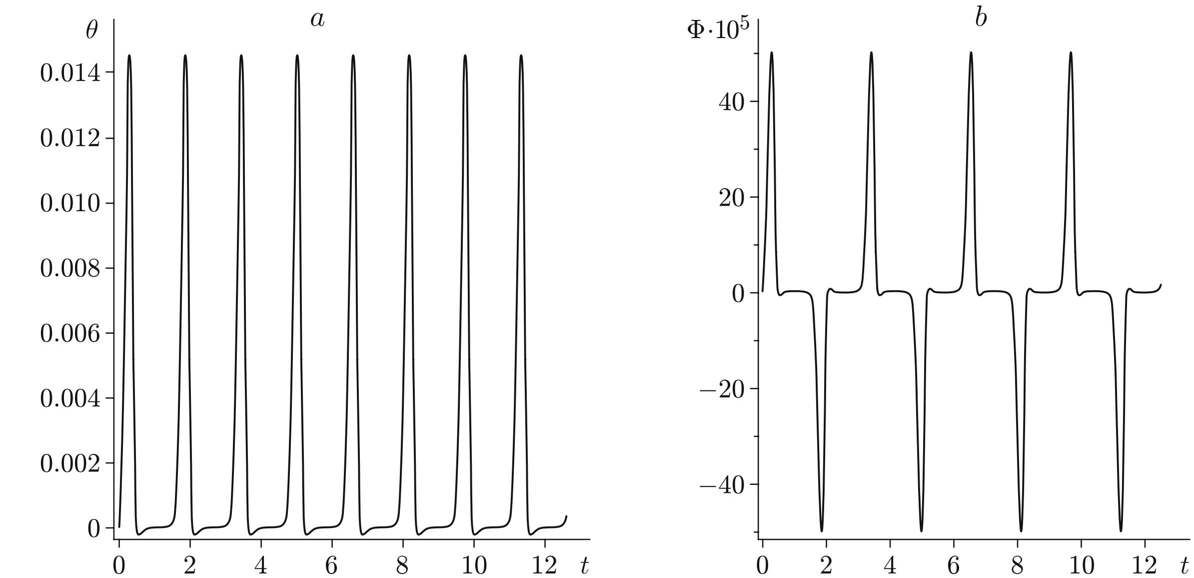

Fig. 3. Synchronous neutral perturbations (class H1) of temperature (a) and magnetic potential (b) for $\omega = 2$, $N = 197{,}600$, $\mathrm{R} = -108{,}200$, $\mathrm{Pr} = 25$, $\chi_0 = 0.12$, $\xi = 4.12$, $k = 8.129$, and $z = 0$.

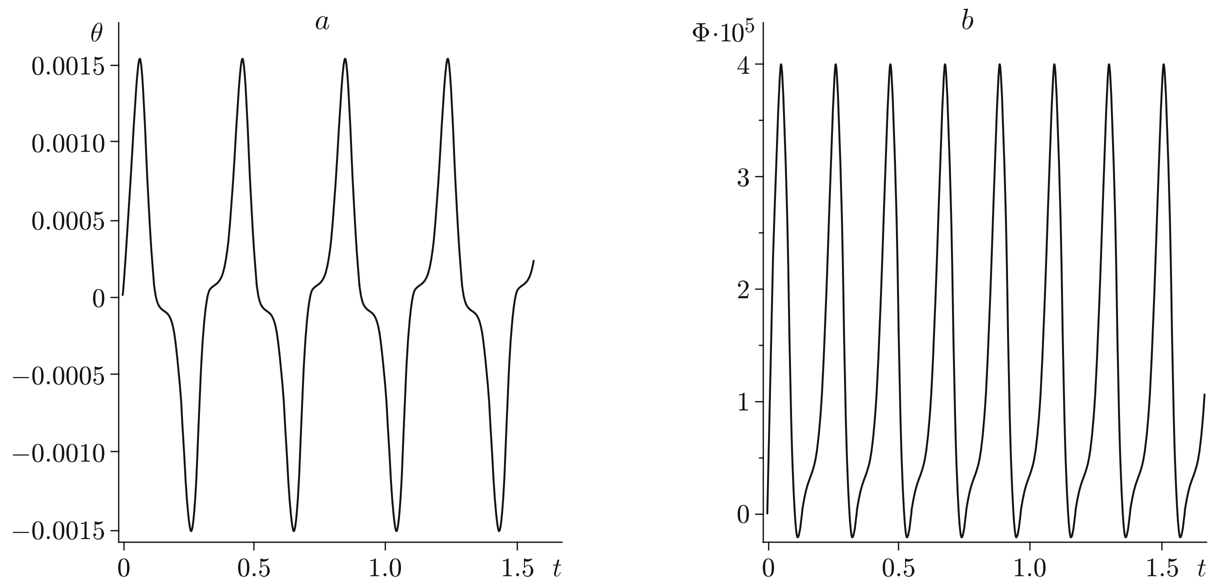

Fig. 4. Synchronous neutral perturbations (class H2) of temperature (a) and magnetic potential (b) for $\omega = 16$, $N = 331{,}250$, $\mathrm{R} = -108{,}200$, $\mathrm{Pr} = 25$, $\chi_0 = 0.12$, $\xi = 4.12$, $k = 8.129$, and $z = 0$.

number ($\mathrm{R} \approx -10^5$; heating from above). At $|\mathrm{R}| \ll 10^5$ ($\mathrm{R} < 0$), the critical magnetic Rayleigh number is almost independent of the action frequency ω, and neutral perturbations possess the properties of perturbations of the synchronous class H1. At $\mathrm{R} > 0$, the system exhibits a similar behavior. More intense heating from above $|\mathrm{R}|$ ($\mathrm{R} < 0$) leads to the emergence of perturbations of the class H2; the zones of instability acquire a shape typical for the parametric resonance.

Figures 3 and 4 show the time evolution of the neutral perturbations of temperature and magnetic potential for the synchronous classes H1 and H2, respectively (the abscissa axis covers the time interval corresponding to four periods of alternation of the external field). It is seen that the class H1 is characterized by the frequency of θ, which is twice higher than the frequency of the external magnetic field [according to Eq. (20), the period of velocity variation coincides with the period of alternation of temperature perturbations]. The potential Φ changes with the external field frequency. Reverse relations are valid for the class H2.

A comparison of Figs. 3a and 4a shows that temperature perturbations θ of the class H1 oscillate around a non-zero mean value, while the time-averaged temperature perturbation θ for the class H2 equals zero. As a consequence, the transition between the classes H1 and H2 is characterized by a jumplike change in the magnitude of the convective heat flux through the layer. Thus, it is possible to control the heat flux through a ferrofluid layer by changing the frequency of the alternating magnetic field.

Conclusions. The influence of an alternating magnetic field on the onset of convection in a horizontal plane-parallel layer of a ferrofluid is studied within the framework of the linear problem of stability. Based on matrix transformations, it is demonstrated that subharmonic perturbations of ferrofluid equilibrium cannot occur if the mean value of the magnetic field is zero. The critical magnetic Rayleigh number N is plotted as a function of the field alternation frequency ω. The calculations show that the parametric resonance can appear only for negative Rayleigh numbers ($\mathrm{R} < 0$), which corresponds to heating from above. For low absolute values of the Rayleigh number ($|\mathrm{R}| \ll 10^5$), the threshold value of N is practically independent of frequency, while the emergence of characteristic resonance "pockets" and alternation of responses of different types in the neighboring regions of convective instability are observed at $\mathrm{R} \approx -10^5$. In the case of an alternating magnetic field, the perturbations have an extremely synchronous character and can be divided into the classes H1 and H2 with different ratios of the periods of oscillations of temperature and magnetic field potential perturbations.

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